Target-In/Target-Out running –Statistical Considerations

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Let $N_{beam}^{in,out}$ be the number of beam particles sampled with the target in and out.
Let $N_{ev}^{in,out}$ be the number of events recorded with the target in and out.
Let $N_{good}^{in,out}$ be the number of good events that survive after all the cuts for the in,out samples.
Both target-in and target-out good events have had all the offline cuts placed on them including vertex constraints.

Let λ be the target interaction length. λ is approximately 0.01.

Then let $N_{ev}^{in,out} = N_{beam}^{in,out}\ f^{in,out}$, where $f^{in,out}$ are the fractions of beam particles that get recorded on to disk. These numbers should be similar in magnitude.
Let $N_{good}^{in,out}=g^{in,out}N_{ev}^{in,out}$.
Since we are dead time limited, the running time in both target in and out cases is dictated by the number of events written to disk.
Let $N_{ev}^{tot}=N_{ev}^{in}+N_{ev}^{out}$. The plan is to optimize the fraction r of target out running for a fixed number of events written to disk.

$N_{ev}^{out}=r\ N_{ev}^{tot}$; and $N_{ev}^{in}=(1-r)N_{ev}^{tot}$

We are trying to determine the cross section which is proportional to the interaction length λ where,

$$l = f^{in}g^{in} - f^{out}g^{out}$$

$s_l^2 = (f^{in})^2 s_{gin}^2 + (f^{out})^2 s_{gout}^2$ where σ^2 denotes variance.

$$g^{in} = \frac{N_{good}^{in}}{N_{ev}^{in}}$$

$$s_{gin}^{2} = \frac{s_{N_{good}^{in}}^{2}}{N_{ev}^{in\,2}} = \frac{N_{good}^{in}}{N_{ev}^{in\,2}} = \frac{g^{in}}{N_{ev}^{in}}$$

$$s_{gout}^{2} = \frac{s_{N_{good}^{out}}^{2}}{N_{ev}^{out\,2}} = \frac{N_{good}^{out}}{N_{ev}^{out\,2}} = \frac{g^{out}}{N_{ev}^{out}}$$

This leads to

$$s_{l}^{2} = \frac{1}{N_{ev}^{tot}}\left(\frac{f^{in\,2} g^{in}}{1-r} + \frac{f^{out\,2} g^{out}}{r}\right)$$

We need to minimize the error on the cross section with respect to r.

$$\frac{ds_{l}^{2}}{dr} = \frac{1}{N_{ev}^{tot}}\left(\frac{f^{in^2} g^{in}}{(1-r)^2} - \frac{f^{out\,2} g^{out}}{r^2}\right) = 0$$

This leads to

$$\frac{r^2}{(1-r)^2} = \frac{f^{out^2} g^{out}}{f^{in\,2} g^{in}} \equiv k^2$$

leading to

$$r = \frac{k}{1+k}$$

$$k = \frac{f^{out}}{f^{in}}\sqrt{\frac{g^{out}}{g^{in}}}$$

Figure 1 shows the quantity r as a function of k^2.

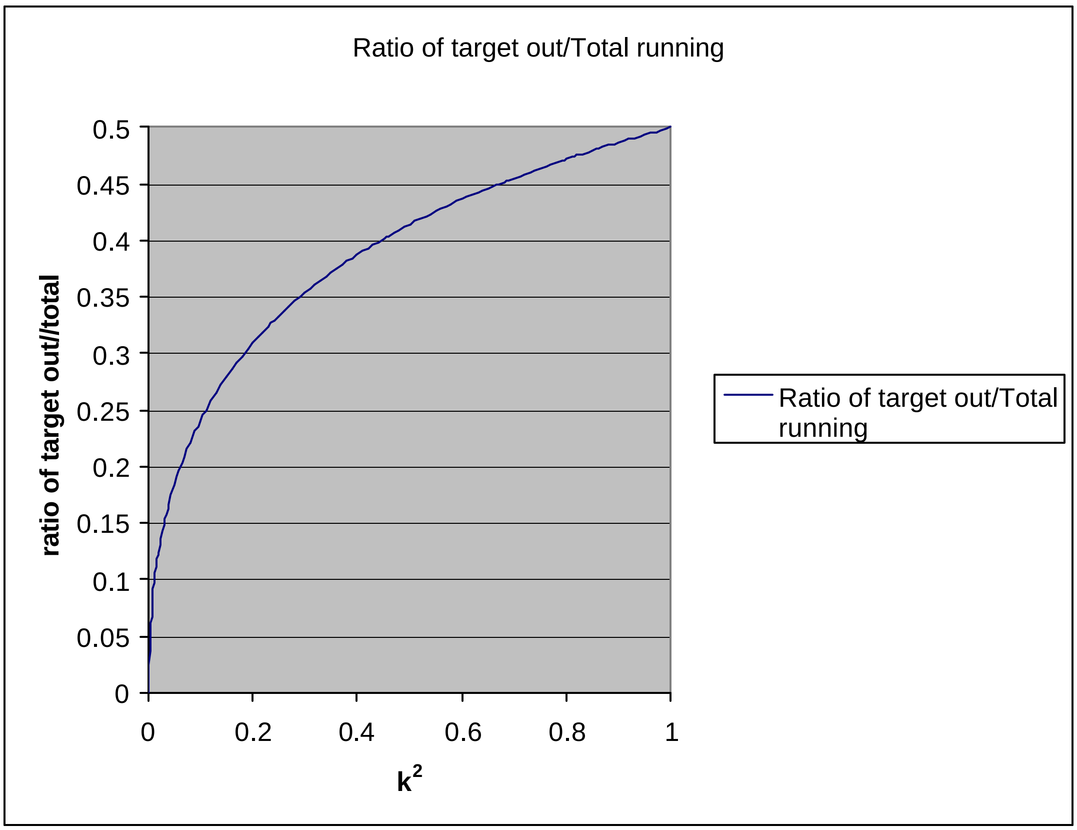

Figure 1 Optimal target-out running as a function of the ratio k^2.

Notice the square root factor actually works against us, since g^{out}/g^{in} is less than unity.

With the optimum value of r, it is easy to show that

$$s_l^2 = \frac{f^{in^2} g^{in}}{N_{ev}^{tot}(1-r)}(1+k) = \frac{f^{in^2} g^{in}}{N_{ev}^{in}}(1+k)$$

i.e 1+k is the factor by which the purely target-in variance $\frac{f^{in^2} g^{in}}{N_{ev}^{in}}$ is inflated due to optimal target-out subtraction.